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# LAW OF VAPOR-BUBBLE GROWTH IN A TUBE IN THE REGION OF LOW PRESSURES

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*Using an analog of a Rayleigh equation the limiting law of vapor-bubble growth in a tube in coolant boiling in the region of very low pressures is obtained.*

The problem of the growth of a vapor slug in coolant boiling in a heated tube was investigated in [1-4]. As is known [5] the process of bubble boiling in the low-pressure region is characterized by low surface density of nucleation sites. Therefore when describing the initial step of the process of boiling we can consider, to a good approximation, the problem of the growth of a spherical vapor bubble on a certain segment of the tube length. The corresponding conjugation conditions [4] should be specified at the boundaries of these neighboring segments. In the limiting case of "very low pressure," the problem of vapor-bubble growth in a single tube with a specified constant pressure at both its outlets (i.e., at the boundaries of the segment) can be considered. As a consequence of the high rates of bubble growth that are characteristic of boiling in the low-pressure region, viscous and gravitational effects will be negligibly small as compared to inertial ones [5]. As is known [6] the dynamics of a spherical vapor bubble in an unbounded volume of liquid is described by the classical Rayleigh equation:

$$\frac{\Delta P}{\rho} = \frac{3}{2}\dot{R}^2 + R\ddot{R}. \tag{1}$$

For the problem of bubble growth in a sufficiently long tube ($l_*/R_0 >> 1$), the Rayleigh equation becomes unacceptable. In [7], its corresponding analog

$$\frac{\Delta P}{\rho} = 2\frac{Rl_*}{R_0^2}(2\dot{R}^2 + R\ddot{R}) \tag{2}$$

is derived for this case. Here $R$ is the bubble radius; $\dot{R} \equiv dR/dt$; $\ddot{R} \equiv d^2R/dt^2$; $t$ is the time; $\rho$ is the liquid density; $\Delta P = P - P_\infty > 0$ is the pressure difference; $P$, $P_\infty$ are the pressures in the bubble and in nominal "cross-sections of the outlet" from the tube, respectively; $R_0$ is the tube radius; $l_*$ is the characteristic length, which is defined by the distances $l_1$ and $l_2$, i.e., the "coordinates of the distance" of the nucleate site under study from the nominal "outlets" from the tube:

$$l_* \equiv \frac{l_1 l_2}{l_1 + l_2}. \tag{3}$$

A theoretical analysis of the process of vapor-bubble growth in an unbounded volume of liquid as applied to the region of very low pressures is performed in [8]. The model of [8] involves the Rayleigh equation, an energy equation for the liquid around the bubble, an approximation of the portion of the saturation curve for the low-pressure region, as well as detailed evaluations of the assumptions made in the analysis. As a result, in [8], the following "limiting" law of bubble growth was obtained:

$$R = \beta_1 F^{3/4}\left(\frac{\lambda' c_p'}{\rho_s'' r}\right)^{1/4} T_s^{1/2} t^{3/4}. \tag{4}$$

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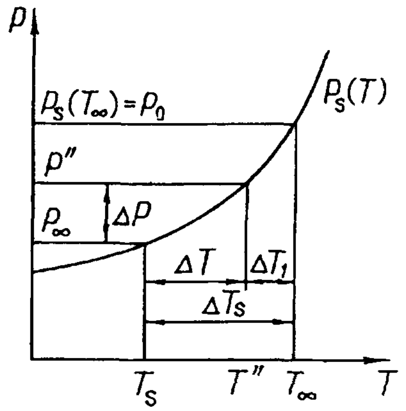


Fig. 1. Relationship of the differences of pressures and temperatures along saturation curve.

Here $\lambda'$ and $c_p'$ are the thermal conductivity and specific heat of the liquid, respectively; $T_s = T_s(P_\infty)$ is the saturation temperature for the pressure in the liquid at infinity; $r$, the specific heat of the phase transition; $\rho_s''$, the vapor density at the saturation temperature $T_s$; $F \equiv (\tilde{R}T_s/r) \simeq 0.1$ is the "Trouton parameter"; $\tilde{R}$ is the individual gas constant; $\beta_1 \sim 1$.

Our work seeks to generalize the analysis of [8] to the case of vapor-bubble growth in a sufficiently long tube $l_*/R_0 >> 1$. The entire course of the reasoning of [8] is completely borrowed but, instead of Rayleigh equation (1), its analog, i.e., Eq. (2), is used. In what follows the basic points of the analysis [8] are briefly represented.

1. The approximation of the saturation curve in the region of low pressures:

$$\Delta P = \frac{\rho_s'' r \Delta T^2}{\tilde{R} T_s^3}. \tag{5}$$

Here $\Delta T = T_\infty - T_s(P_\infty)$; $T_\infty$ is the temperature of the superheated liquid at infinity (in the case under study, at the boundaries of the tube); $T_s$ is the saturation temperature for the boundary pressure $P_\infty$.

2. The heat-flux density $q_R$ at the boundary of the vapor bubble:

$$q_R = \left(\frac{\lambda' c_p' \rho'}{t}\right)^{1/2} \Delta T_1 , \tag{6}$$

where $\Delta T_1 = T_\infty - T''$ is the "liquid at infinity – vapor in the bubble" temperature difference; $T'' = T_s(P'')$ is the temperature of the saturated vapor in the bubble, which varies with time as a consequence of the change in the vapor pressure along the saturation curve in bubble growth.

3. The relationship of the temperature differences $\Delta T$, $\Delta T_1$, and $\Delta T_s$:

$$\Delta T + \Delta T_1 = \Delta T_s . \tag{7}$$

Here $\Delta T$ is the temperature difference reckoned along the saturation curve; $\Delta T_1$ is the working temperature difference, which enters energy equation (6); $\Delta T_s \equiv T_\infty - T_s(P)$ is the total temperature difference (Fig. 1).

4. The heat balance equation for a spherical bubble:

$$q_R = r\rho'' \dot{R} . \tag{8}$$

The use of relations (4)-(8) simultaneously with Rayleigh-equation analog (2) leads to the following "limiting" law of bubble growth in a tube:

$$R = \beta_2 \left( \frac{c' \lambda'}{\rho_s'' r l_*} \right)^{1/5} (T_s R_0)^{2/5} t^{3/5} . \quad (9)$$

Here $\beta_2 \approx 0.1$ with allowance made for the Trouton rule: $r \approx 10\tilde{R}T_s$.

The use of relation (9) for the time of bubble filling of the entire cross-section of the tube ($t = t_0$; $R = R_0$) permits calculation of the velocity of the bubble boundary $\dot{R}_0$ and the "bubble – tube boundary" pressure difference $\Delta P_0$ at the end of the initial step of boiling, i.e., the generation and growth of a spherical vapor bubble:

$$\dot{R}_0 = 1.3 \cdot 10^{-2} U ; \quad (10)$$

$$\Delta P_0 = 6.7 \cdot 10^{-4} (l_* / R_0) \rho' U^2 . \quad (11)$$

Here $U$ is the velocity scale, which is determined in the following manner:

$$U \equiv \left( \frac{c_p' \lambda'}{\rho'' r l_*} \right)^{1/3} T_s^{2/3} . \quad (12)$$

Expressions (11) and (12) can be used as initial conditions to calculate the next step of liquid boiling in a tube in the region of very low pressures – the problem of vapor-slug formation and growth [4]. The applicability limits for the above law of the growth of a vapor bubble in a tube are governed by the realization conditions of [8] for the "limiting" computational scheme:

$$\frac{\Delta T_1}{T_s} \geq 0.1 ; \quad \frac{\rho' c' \Delta T_1}{r \rho''} \geq 600 . \quad (13)$$

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## NOTATION

$R$, bubble radius; $\dot{R} \equiv dR/dt$; $\ddot{R} = d^2R/dR^2$; $t$, time; $\rho$, liquid density; $\lambda$, thermal conductivity of the liquid; $c_p$, specific heat of the liquid; $\Delta P = P'' - P_\infty$, pressure difference; $P''$, pressure in the bubble; $P_\infty$, pressure at the outlets from the tube; $R_0$, tube radius; $l_1$, $l_2$, distances from the boiling center to the outlets from the tube; $l_* = l_1 l_2/(l_1 + l_2)$, characteristic length; $T_s \equiv T_s(P_\infty)$, saturation temperature for the pressure in the liquid at infinity; $r$, specific heat of the phase transition; $\rho_s''$, vapor density that is taken at the saturation temperature $T_s$; $F \equiv (\tilde{R}T_s/r) \simeq 0.1$, "Trouton parameter"; $\tilde{R}$, individual gas constant; $\Delta T = T_\infty - T_s(P_\infty)$, temperature difference reckoned along the saturation curve; $T_\infty$, temperature of the superheated liquid at the outlets from the tube; $q_R$, heat-flux density at the boundary of the vapor bubble; $\Delta T_1 = T_\infty - T''$, "liquid at infinity – vapor in bubble" temperature difference; $T'' = T_s(P'')$, temperature of the saturated vapor in the bubble; $\Delta T_s \equiv T_\infty - T_s(P)$, total temperature difference; $U$, velocity scale. Superscripts: $''$ conditions in the vapor phase; $\dot{}$, $\ddot{}$ first and second derivatives with respect to time, respectively. Subscripts: 0, conditions on the tube wall; $\infty$, conditions at the outlet from the tube; s, saturation conditions; $R$, conditions at the boundary of the vapor bubble.

## REFERENCES


1. W. D. Ford, H. K. Fauske, and S. G. Bankoff, Int. J. Heat Mass Transfer, 14, No. 1, 133-139 (1971).
2. V. F. Prisnyakov, Inzh.-Fiz. Zh., 25, No. 3, 440-444 (1973).
3. V. N. Serebryanskii, Teplofiz. Vys. Temp., 26, No. 4, 767-773 (1988)

4. S. I. Lezhnin and S. P. Aktershev, Transactions of the 1st Russian National Conference on Heat Transfer, Vol. 5. Evaporation and Condensation [in Russian], Moscow (1994), pp. 97-102.
5. V. V. Yagov, "Approximate theory of heat transfer and boiling crises of liquids," Author's Abstract of Doctor's Dissertation in Technical Sciences, Moscow (1988).
6. G. Lamb, Hydrodynamics [Russian translation], Moscow (1947).
7. Yu. B. Zudin, Inzh.-Fiz. Zh., 63, No. 1, 28-31 (1992).
8. V. V. Yagov, Teplofiz. Vys. Temp., 26, No. 2, 335-341 (1988).